

MATH 261: Calculus I

Exam 4

NAME: _____

Instructions. This exam is “closed-book”. You may not use notes or electronic devices of any kind. Show work and circle your final answers. Give exact answers— do not approximate unless asked.

1. (2 points) Complete the sentence. By definition, a function $f(x)$ is *differentiable* at $x = a$ if

2. (2 points) Complete the sentence. By definition, a function $f(x)$ is *continuous* at $x = a$ if

3. (2 points) Find the derivative of $y = \sqrt{3x + \sqrt{x}}$ at $x = 1$.

EXAM B

4. (2 points) Suppose $g(x) = -f(1 - 2x)$. If $f'(2) = 5$, what is $g'(-1/2)$.

5. (2 points) Suppose $f(x)$ is differentiable, and $g(x) = [f(-x)]^2$. If $f'(x) > 0$ then $g(x)$ is

- (a) increasing (b) decreasing (c) varies

6. (2 points) Compute the limit.

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1}$$

7. (2 points) Compute the limit.

$$\lim_{h \rightarrow 0} \frac{\tan(\pi/4 + h) - 1}{h}$$

EXAM B

8. (3 points) A boy is throwing water balloons straight down from the top of a 100 ft building. If he throws a balloon at 20 ft/sec, how long will it take to hit the ground? Round your answer to the nearest tenth of a second. (Assume acceleration due to gravity is 32 ft/s^2 , no air resistance, and $\sqrt{17} \approx 4.12$.)

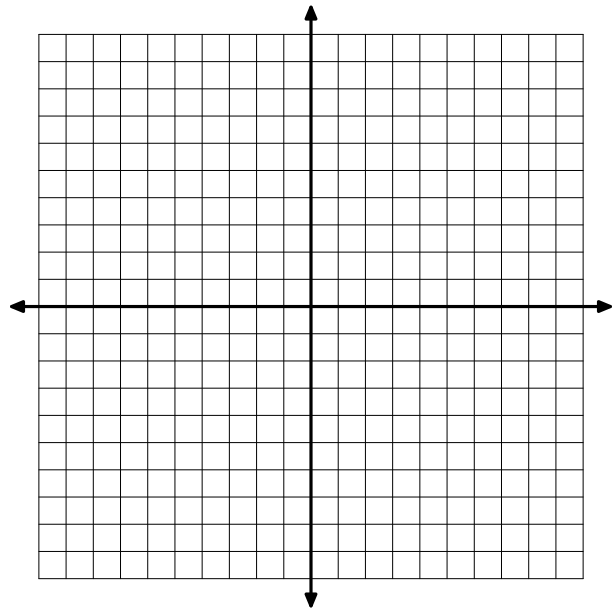
9. (4 points) Find the slope of the tangent line to $x \sin y = 1$ at $(2, \pi/6)$.

10. (2 points) A spherical balloon is inflated at $10 \text{ cm}^3/\text{s}$. What is the rate of change of the radius when the volume is 100 cm^3 ? ($V = \frac{4}{3}\pi r^3$.)

EXAM B

11. (2 points) Use a linear approximation or two iterations of Newton's Method (your choice) to estimate $\sqrt{9998}$.

12. (5 points) Sketch a graph of $y = \frac{4x^2}{x^2 + 3}$. Be sure to give the asymptotes, relative extrema, and inflection points (if any). Scale the grid appropriately to fit the graph.



EXAM B

13. Compute the definite or indefinite integrals. Circle your final answers.

(a) (3 points) $\int x\sqrt{x^2 + 1} dx$

(b) (3 points) $\int \frac{3}{(2y + 1)^5} dy$

(c) (3 points) $\int_0^{\pi/3} \frac{\sin \theta}{\cos^2 \theta} d\theta$

EXAM B

14. (4 points) Find the area bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$.

15. (4 points) Find the volume of the solid obtained by rotating about the y -axis the region bounded by $y = x^3$, $x = 1$, and $y = 0$.

EXAM B

16. (3 points) A 30 ft wire is hanging from the top of a building. The wire weighs 1 lb/ft. How much work does it take to lift the wire to the top of the building?

17. (3 points) If f is continuous on $[a, b]$, the *average value* of f over $[a, b]$ is defined as

$$\frac{\int_a^b f(x) dx}{b - a}.$$

Find the average value of

$$f(x) = \frac{x}{\sqrt{x-1}}$$

over $[2, 3]$.