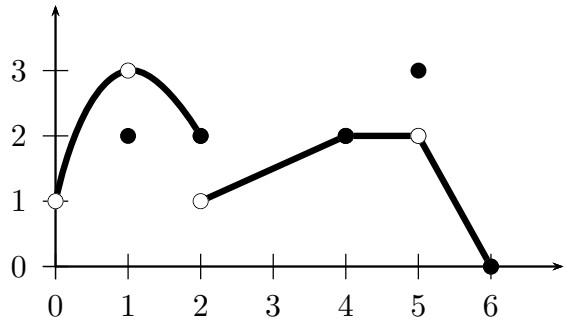
MATH 261: Calculus I 2014-09-25

Exam One

NAME: _____

Instructions. This exam is "closed-book". You may not use notes or electronic devices of any kind. Show work and circle your final answers.

Use the graph of $y = f(x)$ below to answer questions 1–4. Write nothing if your answer is either "undefined" or "does not exist".



1. (4 points) Find the following function values.
   (a) $f(1)$ _____ (b) $f(2)$ _____
   (c) $f(4)$ _____ (d) $f(5)$ _____

2. (6 points) Find the following limits.
   (a) $\lim_{x\to 1} f(x)$ _____ (b) $\lim_{x\to 2^-} f(x)$ _____
   (c) $\lim_{x\to 2^+} f(x)$ _____ (d) $\lim_{x\to 4} f(x)$ _____
   (e) $\lim_{x\to 2} f(x)$ _____ (f) $\lim_{x\to 5} f(x)$ _____

3. (4 points) True or False. (Write **T** or **F**.) $f(x)$ is continuous at
   (a) $x = 1$ _____ (b) $x = 2$ _____
   (c) $x = 4$ _____ (d) $x = 5$ _____

4. (4 points) True or False. (Write **T** or **F**.) $f(x)$ is differentiable at
   (a) $x = 1$ _____ (b) $x = 2$ _____
   (c) $x = 4$ _____ (d) $x = 5$ _____

5. (15 points) Compute the limits. If a limit does not exist, explain. Note that "does not exist" is an invalid response if the limit is $\pm\infty$.

(a) $\lim_{x\to 2^+} f(x)$ where $f(x) = \begin{cases} 3-2x & \text{if } x < 2 \\ x^2+1 & \text{if } x > 2 \end{cases}$

(b) $\lim_{y\to 25} \dfrac{25-y}{\sqrt{y}-5}$

(c) $\lim_{x\to 0^-} \dfrac{|x|}{x}$

(d) $\lim_{w\to -1} \dfrac{w^3+w^2-4w-4}{w^2-1}$

(e) $\lim_{x\to 1} \dfrac{\dfrac{1}{x}-1}{x-1}$

6. (3 points) Use the Squeeze Theorem to show that

$$\lim_{x \to 0^+} x \sin(1/x) = 0.$$

7. (3 points) Complete the following sentence. *A function $f(x)$ is continuous at $x = a$ if*

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8. (3 points) Show that the function $f(x) = x^5 - x + 1$ has a zero in the interval $[-2, 0]$.

9. (5 points) For $f(x) = \dfrac{x+1}{x}$ compute directly

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(Note: Your final answer should be $\frac{-1}{x^2}$.)

10. (3 points) Using the result of the previous problem, find the equation of the tangent line to $f(x) = (x+1)/x$ at $x = 1$.

11. (3 points) We say $\lim_{x \to a} f(x) = \infty$ if given any $N > 0$ there is a $\delta > 0$ such that $f(x) > N$ whenever $0 < |x - a| < \delta$. Using this definition, prove

$$\lim_{x \to 0} \frac{1}{x^2} = \infty.$$